HYBRID APPROACH BASED ON TENSOR DECOMPOSITION FOR MODEL ORDER REDUCTION APPLIED TO MOTOR DIAGNOSIS

Ze Guo[1,2,3], Zuqi Tang[1]

[1]Univ. Lille, Arts et Métier Institute of Technology, Centrale Lille, Junia, ULR 2697-L2EP, F-59000 Lille, France
[2]Institute of Electrical Engineering, Chinese Academy of Sciences, Beijing 100190, China
[3]University of Chinese Academy of Sciences, Beijing 100190, China
zuqi.tang@univ-lille.fr

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Abstract

In this work, a hybrid model order reduction (MOR) approach is proposed for nonlinear multi-parameter magnetostatic problems applied to motor diagnostics, considering various operating conditions and fault scenarios. The proposed approach leverages tensor decomposition to construct both the reduced basis and the nonlinear term, demonstrating superior performance compared to classical methods.

1 Introduction

Reliable motor operation is critical, driving an increased emphasis on online monitoring and fault diagnosis within the digital twin framework. The development of a digital twin diagnostic model for electric motors requires comprehensive data spanning diverse operating conditions and fault scenarios, such as eccentricity, demagnetization, short circuits, and open circuits [1]. This demand underscores the need for numerical simulation of multi-parameter problems to generate accurate and cost-effective data.

However, repeated high-fidelity numerical simulations, such as those using finite element methods, involving multiple parameters and nonlinear iterations, are computationally expensive. To mitigate this computational burden, projection-based model order reduction (MOR) techniques, such as Proper Orthogonal Decomposition (POD), have been widely adopted. Nevertheless, POD-based MOR is often ineffective for nonlinear multi-parameter problems because the reduced-order space generated by POD can become excessively large due to its limited ability to capture the parameter-specific characteristics of the system [2].

To address the challenges arising in multi-parameter cases, a hybrid approach based on tensor decomposition (TD) techniques is proposed to construct a both problem-dependent and parameter-specific MOR basis, as reported in our recent work [3]. In this approach, nonlinear terms, such as magnetic reluctivity, are approximated at the mesh element level using snapshot interpolation, effectively reducing the nonlinear problem to a linear one. However, the interpolation process for these nonlinear terms can become computationally expensive and memory-intensive during the online stage as the parameter dimensionality increases, limiting the applicability of this method to higher-dimensional parameter problems.

In this work, by leveraging the tensor structure, the interpolation process is improved and strengthened, requiring less memory and reducing online computational time. This renders our proposed approach an efficient hybrid MOR method. Constructing both the MOR bases and the nonlinear terms independently yet simultaneously provides the potential to handle multi-parameter problems in high dimension effectively.

2 Methodology

Considering the diverse operating conditions and fault scenarios of the motor, the corresponding nonlinear magnetostatic problem with multiple parameters, represented by a D-dimensional parameter vector $\mathbf{p} = (p_1, p_2, \dots, p_D)$, is expressed as:

$$\mathbf{curl}(\boldsymbol{\nu}(\mathbf{B})\mathbf{curl}\mathbf{A}) = \mathbf{J}_s + \mathbf{curl}\mathbf{M}, \qquad (1)$$

where $\boldsymbol{\nu}$ represents the magnetic reluctivity, $\mathbf{B}$ the magnetic flux density, $\mathbf{A}$ the magnetic vector potential such that $\mathbf{B} = \mathbf{curl}\mathbf{A}$, $\mathbf{J}_s$ the imposed current density, $\mathbf{M}$ the magnetization vector of permanent magnet. This can be simplified to the following system:

$$\mathbb{K}(\boldsymbol{u}, \mathbf{p})\boldsymbol{u}(\mathbf{p}) = \mathbf{F}(\mathbf{p}), \qquad (2)$$

where $\mathbb{K}(\boldsymbol{u}, \mathbf{p})$ represents the $\mathbf{p}$ dependent system matrix, $\boldsymbol{u}(\mathbf{p})$ represents the discretized form of the unknowns in the system, and $\mathbf{F}(\mathbf{p})$ is the right-hand side vector resulting from the discretization of the source term.

This nonlinear system can be solved using nonlinear schemes such as Newton-Raphson or the Fixed-Point method. For parametric problems, when solving repeatedly with different values of $\mathbf{p}$, to avoid the costly nonlinear iteration process, the interpolation technique for $\boldsymbol{\nu}$ can be used in practice. The equivalent problem is expressed as:

$$\mathbb{K}(\boldsymbol{\nu}(\mathbf{p}), \mathbf{p})\boldsymbol{u}(\mathbf{p}) = \mathbf{F}(\mathbf{p}). \qquad (3)$$

The advantage of this approach is that only a linear problem needs to be solved. However, it requires $\boldsymbol{\nu}(\mathbf{p})$, which is typically obtained through table interpolation. In practice, the computational cost and storage for the $\boldsymbol{\nu}$ term can be expensive, particularly when the dimension of the parameter space becomes large.

For the MOR, applying the POD method, with a reduced basis $\mathbb{U}$ satisfying $\mathbb{U}^{\mathrm{T}}\mathbb{U}=\mathbb{I}$, the classical reduced model reads:

$$\mathbb{U}^{\mathrm{T}}\mathbb{K}(\mathbf{v}(\mathbf{p}),\mathbf{p})\mathbb{U}\mathbb{U}^{\mathrm{T}}\boldsymbol{u}(\mathbf{p})=\mathbb{U}^{\mathrm{T}}\mathbf{F}(\mathbf{p}). \tag{4}$$

In our previous work, TD-based MOR is proposed to construct the reduced basis $\mathbb{U}_{\mathrm{TD}}(\mathbf{p})$ with the term $\mathbf{v}(\mathbf{p})$ obtained by classical interpolation. Here, we aim to use the tensor structure to alleviate the computational burden corresponding to the construction of $\mathbf{v}(\mathbf{p})$.

With our proposed TD based approach, we need to construct two terms, namely $\mathbb{U}_{\mathrm{TD}}(\mathbf{p})$ for $\mathbb{U}$ and $\mathbf{v}_{\mathrm{TD}}(\mathbf{p})$ for $\mathbf{v}(\mathbf{p})$ arsing in (4).

2.1 Construction of $\mathbb{U}_{\mathrm{TD}}$ by TDMOR

Similar to the classical POD method, we need to collect the snapshots, by sweeping the parameter $\mathbf{p}$, to get $\boldsymbol{u}(p_1^{j_1},\dots,p_D^{j_D})$, with $(p_i^j)_{j=1,\dots,n_i}$ and $1\le i\le D$. With the tensor structure, a $D+1$ dimension tensor $\boldsymbol{\Phi}_{\boldsymbol{u}}$, of size $m\times n_1\times n_2\times\dots\times n_D$, is constructed as:

$$\boldsymbol{\Phi}_{\boldsymbol{u}}(:,j_1,\dots,j_D)=\boldsymbol{u}(p_1^{j_1},\dots,p_D^{j_D}). \tag{5}$$

With the Tensor decomposition, a low rank approximation $\widetilde{\boldsymbol{\Phi}}_{\boldsymbol{u}}$ of the full tensor $\boldsymbol{\Phi}_{\boldsymbol{u}}$ can be obtained:

$$\widetilde{\boldsymbol{\Phi}}_{\boldsymbol{u}}=\sum_{j=1}^{\tilde{m}}\sum_{q_1=1}^{\tilde{n}_1}\dots\sum_{q_D=1}^{\tilde{n}_D}(\mathbf{C}_{\boldsymbol{u}})_{j,q_1,\dots,q_D}\boldsymbol{w}_{\boldsymbol{u}}^{j}\circ\boldsymbol{\sigma}_{\boldsymbol{u};1}^{q_1}\circ\dots\circ\boldsymbol{\sigma}_{\boldsymbol{u};D-1}^{q_{D-1}}\circ\boldsymbol{\sigma}_{\boldsymbol{u};D}^{q_D}, \tag{6}$$

where $\widetilde{m},\tilde{n}_1,\dots\tilde{n}_D$ are the compressed dimension much smaller than $m,n_1,\dots,n_D$, implying that the core tensor $\mathbf{C}_{\boldsymbol{u}}$ has a small size.

TDMOR constructs parameter-specific reduced bases $\mathbb{U}_{\mathrm{TD}}(\mathbf{p})$ in two stages. Specifically, at the offline stage, a global orthogonal basis $\mathbb{U}_{\boldsymbol{u};\mathrm{g}}$ and a series of matrices $\mathbb{S}_{\boldsymbol{u};k}$ with $1\le k\le D-1$, that encodes information on the relationship between various parameters, can be obtained according to (6).

$$\mathbb{U}_{\boldsymbol{u};\mathrm{g}}=\left[\boldsymbol{w}_{\boldsymbol{u}}^{\mathbf{1}},\dots,\boldsymbol{w}_{\boldsymbol{u}}^{\tilde{m}}\right]\in R^{m\times\tilde{m}}, \tag{7}$$

$$\mathbb{S}_{\boldsymbol{u};k}=\left[\boldsymbol{\sigma}_{\boldsymbol{u};k}^{1},\dots,\boldsymbol{\sigma}_{\boldsymbol{u};k}^{\tilde{n}_k}\right]^{T}\in R^{\tilde{n}_k\times n_k}, \tag{8}$$

At the online stage, a parameter specific core matrix $\mathbb{C}_{\boldsymbol{u}}(\mathbf{p})$ can be obtained with the core tensor $\mathbf{C}_{\boldsymbol{u}}$, matrices $\mathbb{S}_{\boldsymbol{u};k}$ and the following extraction-interpolation procedure.

$$\mathbb{C}_{\boldsymbol{u}}(\mathbf{p})=\mathbf{C}_{\boldsymbol{u}}\times_2\left(\mathbb{S}_{\boldsymbol{u};1}\boldsymbol{e}^1(\mathbf{p})\right)\dots\times_D\left(\mathbb{S}_{\boldsymbol{u};D-1}\boldsymbol{e}^{D-1}(\mathbf{p})\right), \tag{9}$$

where $\boldsymbol{e}^k$ is the coordinate vector at the k-th dimension of the parameter $\mathbf{p}$ obtained by Lagrangian interpolation. The singular value decomposition (SVD) is only needed to apply to the small size core matrix $\mathbb{C}_{\boldsymbol{u}}(\mathbf{p})$:

$$\mathbb{C}_{\boldsymbol{u}}(\mathbf{p})=\mathbb{U}_{\boldsymbol{u};\mathrm{c}}(\mathbf{p})\Sigma_{\boldsymbol{u};\mathrm{c}}(\mathbf{p})\mathbb{V}_{\boldsymbol{u};\mathrm{c}}^{\mathrm{T}}(\mathbf{p}). \tag{10}$$

Finally, a reduce basis $\mathbb{U}_{\mathrm{TD}}(\mathbf{p})$ with both problem-dependent and parameter-specific can be obtained:

$$\mathbb{U}_{\mathrm{TD}}(\mathbf{p})=\mathbb{U}_{\boldsymbol{u};\mathrm{g}}\mathbb{U}_{\boldsymbol{u};\mathrm{c}}(\mathbf{p}). \tag{11}$$

2.2 Construction of $\mathbf{v}_{\mathrm{TD}}$ by TD

Similar to (5), a tensor for the snapshot of $\mathbf{v}(\mathbf{p})$ is constructed as:

$$\boldsymbol{\Phi}_{\mathbf{v}}(:,j_1,\dots,j_D)=\mathbf{v}(p_1^{j_1},\dots,p_D^{j_D}). \tag{12}$$

Similar, Tensor decomposition, as described in (6), can be applied to $\boldsymbol{\Phi}_{\mathbf{v}}$ at offline to get a global orthogonal basis $\mathbb{U}_{\mathbf{v};\mathrm{g}}$ and a series of matrices $\mathbb{S}_{\mathbf{v};i}$, such that:

$$\mathbb{U}_{\mathbf{v};\mathrm{g}}=\left[\boldsymbol{w}_{\mathbf{v}}^{\mathbf{1}},\dots,\boldsymbol{w}_{\mathbf{v}}^{\tilde{m}}\right]\in R^{m\times\tilde{m}}, \tag{13}$$

$$\mathbb{S}_{\mathbf{v};i}=\left[\boldsymbol{\sigma}_{\mathbf{v};i}^{1},\dots,\boldsymbol{\sigma}_{\mathbf{v};i}^{\tilde{n}_i}\right]^{\mathrm{T}}\in R^{\tilde{n}_i\times n_i},\quad i=1,\dots,D. \tag{14}$$

Then at the online stage, the core matrix $\mathbb{C}_{\mathbf{v}}(\mathbf{p})$ for $\mathbf{v}(\mathbf{p})$ can be obtained with extraction-interpolation procedure:

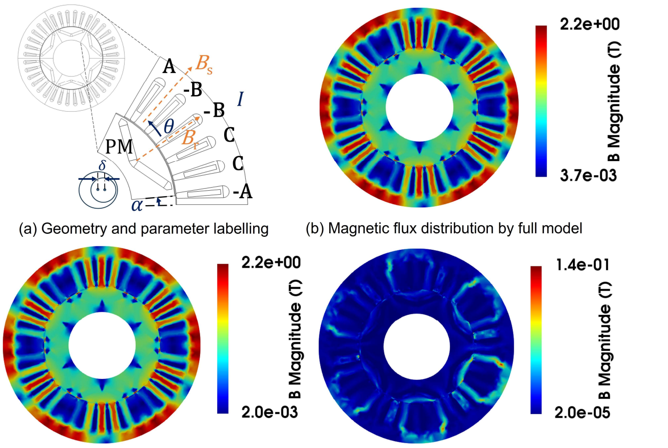
$$\mathbb{C}_{\mathbf{v}}(\mathbf{p})=\mathbf{C}_{\mathbf{v}}\times_2\left(\mathbb{S}_{\mathbf{v};1}\boldsymbol{e}^1(\mathbf{p})\right)\dots\times_D\left(\mathbb{S}_{\mathbf{v};D-1}\boldsymbol{e}^{D-1}(\mathbf{p})\right)\times_{D+1}\left(\mathbb{S}_{\mathbf{v};D}\boldsymbol{e}^{D}(\mathbf{p})\right). \tag{15}$$

Subsequently, $\mathbf{v}_{\mathrm{TD}}(\mathbf{p})$ can be constructed with:

$$\mathbf{v}_{\mathrm{TD}}(\mathbf{p})=\mathbb{U}_{\mathbf{v};\mathrm{g}}\mathbb{C}_{\mathbf{v}}(\mathbf{p}). \tag{16}$$

3 Preliminary Numerical Results

A preliminary result of the proposed approach is illustrated in Fig. 1, invoking an example of electric motor with eccentricity and a total of 4 parameters, which shows a good performance. More detail discussion will be provided in full paper.



(a) Geometry and parameter labelling (b) Magnetic flux distribution by full model

(c) Magnetic flux distribution by MOR model (d) Error of the magnetic flux distribution

Fig. 1. Preliminary result of the propose approach.

References

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